

Optimal Distributed Power Allocation for Decode-and-Forward Relay Networks

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Abstract

This paper presents a fully distributed power allocation algorithm for decode-and-forward (DF) relay networks with a large number of sources, relays, and destination nodes. The well known mathematical decomposition based distributed optimization techniques cannot directly be applied to DF relay networks, because the achievable rate of DF relaying is not strictly concave, and thus the local power allocation subproblem may have non-unique solutions. We resolve this non-strict concavity problem by using the idea of proximal point method, which adds some quadratic terms to make the objective function strictly concave. While traditional proximal point methods require a two-layer nested iteration structure, our proposed algorithm has a single-layer iteration structure, which is desirable for on-line implementation. Moreover, our algorithm only needs local information exchange among the source, relay, and destination nodes of each DF relay link, and can easily adapt to variations of network size and topology. In this paper, we establish the convergence and optimality of our fully distributed single-layer iterative algorithm. Numerical results are provided to illustrate the benefits of our proposed algorithm.

Index terms— Decode-and-forward, distributed power allocation, wireless relay network.

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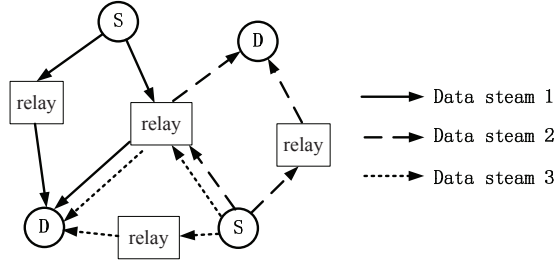


Fig. 1. System model.

I. INTRODUCTION

Cooperative relaying has recently received a lot of attention as a promising technique to improve the throughput, coverage, and reliability of wireless networks [1], [2]. Specifically, the decode-and-forward (DF) relay strategy has been advocated by several standard organizations for next generation wireless networks [3]–[5]. In this strategy, the relay node decodes the source node’s transmission message, then forwards the recovered message to the destination. Recent wireless applications of DF relaying include sensor networks [6], ad-hoc networks [7], cellular networks [8], user-provided networks [9], offshore networks [10], and satellite networks [11], etc.

Since power is a crucial resource in wireless networks, a number of studies have investigated the allocation of power resource for DF relay networks. They have shown that optimal power allocation can achieve significant performance improvement for DF relay networks with a single data stream between the source and destination nodes [12]–[22]. However, optimal power allocation becomes much more challenging for DF relay networks with many data streams, because each data stream may cooperate with several relay nodes. Furthermore, these relay nodes can serve other data streams, as shown in Fig 1. Hence, different data streams in the network are intertwined with each other, which makes the power allocation problem much more complicated. Centralized power allocation for relay networks was studied in [23]–[26], which requests signaling mechanisms to gather the channel state information (CSI) of all the wireless links at a central control node. However, such mechanisms are difficult to implement in practice and will especially not work well when the network size is large.

Recently, a great deal of research efforts have focused on distributed power allocation for DF

relay networks: Traditional two-hop relaying was considered in [27]–[29], which ignores the source-destination wireless channel, and thus achieves a lower data rate. In [30]–[32], several conservative assumptions of the achieve data rate of DF relaying were made to simplify the distributed power allocation problem and make it tractable. Optimal power allocation of the relay nodes was studied in [23]–[26], [33] without considering power allocation at the source node. Optimal distributed power allocation for DF relay networks has remained a difficult and crucial problem.

Mathematical decomposition techniques have been effectively used in multi-hop wireless networks to achieve joint optimal power allocation results (e.g., see [34] and the references therein). However, such techniques cannot be applied directly to DF relay networks — the local power allocation subproblems may have many optimal solutions, because the achievable rate of DF relaying is not strictly concave with respect to the transmission power. Since no global network information is available to the local power allocation subproblems, it is quite difficult to find a global feasible solution among all the locally optimal solutions [35], [36].

One promising method to address this non-strict concavity problem is the *proximal point method* [37], which adds strictly concave terms to the achievable rate function without affecting the optimal solution. However, typical proximal point algorithms require a two-layer nested iteration structure, where each outer-layer update can proceed only after the inner-layer iterations converge [37]. Such a structure is not suitable for distributed implementation, because it is difficult to decide in a distributed manner when the inner-layer iterations can stop.

This paper investigates distributed power allocation for DF relay networks with a large number of sources, relays, and destination nodes. Each source node may transmit to several destinations through the assistance of several relay nodes. Meanwhile, each relay node can serve several data streams (see Fig. 1). We assume that each of the source and relay nodes has an individual transmission power constraint. We propose a fully distributed algorithm, which jointly optimizes the power allocation of the source and relay nodes. The proposed algorithm has the following attractive features: Firstly, the algorithm has a single-layer iteration structure, which is desirable for on-line implementation. Secondly, this algorithm only needs local information exchange among the source, relay, and destination nodes of each DF relay link, and thus can easily adapt to variations of network size and topology. Finally, the convergence and optimality of this algorithm is rigorously established without using any conservative data rate of DF relaying.

The proposed power allocation algorithm is motivated by the work in [38], where a single-layer proximal point algorithm was proposed for multi-path routing problems. However, the structure of the objective function in that work is very different from our work that deals with DF relay networks. Hence, a substantially new proof methodology is required to show convergence in our context, which is one of the major contributions of this paper.

The remaining parts of this paper are organized as follows: In Section II, we present the system model and the formulation of power allocation problem. In Section III, we describe our distributed power allocation algorithm. Numerical results are shown in Section IV, and we conclude in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a DF relay network with N source/destination nodes, denoted by the set $\mathcal{N} = \{1, 2, \dots, N\}$, and J relay nodes, represented by the set $\mathcal{J} = \{1, 2, \dots, J\}$. Each source-destination data stream in the network is denoted as $m = (s, d)$ with $s, d \in \mathcal{N}$. The set of all data streams is denoted by $\mathcal{M} \subseteq \{(i, j) | i, j \in \mathcal{N}, i \neq j\}$. The m th data stream is assisted by $J(m)$ relay nodes, which are represented by the set $\mathcal{J}(m) \subseteq \mathcal{J}$. We assume that the direct transmission (DT) link and DF relay links of each data stream operate over orthogonal wireless channels as in [24], [33].

In practice, wireless networks operate in a half-duplex mode, which means that they transmit and receive signals over different time/frequency channels [12], [14], [33]. Owing to this, the DF relay process consists of 2 phases: In Phase 1, the source node transmits a message to the relay and destination nodes. The relay node decodes its received message, while the destination stores its received signal for later decoding. In Phase 2, the relay node forwards the recovered message to the destination. The destination combines its received signals in two phases to decode the source node's message [2]. Let $h_m^{s,d}$ denotes the complex channel coefficient of the source-destination wireless link of data stream m , $h_{mj}^{s,r}$ and $h_{mj}^{r,d}$ denote the complex channel coefficients of the source-relay and relay-destination links of the DF relay link composed by data stream m and relay node j .

The spectrum efficiency of the DT link of data stream m is given by the capacity of Gaussian channel, i.e.,

$$R_m^{DT} = \theta_m^{DT} \log_2 \left(1 + \frac{P_m^s |h_m^{s,d}|^2}{\theta_m^{DT} N_0 W} \right) = \theta_m^{DT} \log_2 \left(1 + \frac{P_m^s g_m^{s,d}}{\theta_m^{DT}} \right), \quad (1)$$

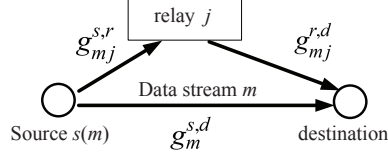


Fig. 2. Illustration of decode-and-forward relay strategy.

where $P_m^s \geq 0$ is transmission power of the source node, θ_m^{DT} is the corresponding proportion of time/frequency channel resource, W is the total amount of available channel resource, N_0 is power spectral density of the Gaussian noise at each receiver, and $g_m^{s,d} \triangleq \frac{|h_m^{s,d}|^2}{N_0 W}$ characterizes the quality of the source-destination wireless channel of the m th data stream, as shown in Fig. 2. The spectrum efficiency achieved by the DF relay link composed by data stream m and relay node j can be described as [2]:

$$\begin{aligned}
 R_{mj}^{DF} &= \frac{\theta_{mj}^{DF}}{2} \min \left\{ \log_2 \left(1 + \frac{2P_{mj}^s |h_{mj}^{s,r}|^2}{\theta_{mj}^{DF} N_0 W} \right), \right. \\
 &\quad \left. \log_2 \left[1 + \frac{2(P_{mj}^s |h_m^{s,d}|^2 + P_{mj}^r |h_{mj}^{r,d}|^2)}{\theta_{mj}^{DF} N_0 W} \right] \right\} \\
 &= \frac{\theta_{mj}^{DF}}{2} \min \left\{ \log_2 \left(1 + \frac{2P_{mj}^s g_{mj}^{s,r}}{\theta_{mj}^{DF}} \right), \right. \\
 &\quad \left. \log_2 \left[1 + \frac{2(P_{mj}^s g_m^{s,d} + P_{mj}^r g_{mj}^{r,d})}{\theta_{mj}^{DF}} \right] \right\}, \tag{2}
 \end{aligned}$$

where $P_{mj}^s, P_{mj}^r \geq 0$ are transmission powers of the source and relay nodes, θ_{mj}^{DF} is the corresponding proportion of time/frequency channel resource, $g_{mj}^{s,r} \triangleq \frac{|h_{mj}^{s,r}|^2}{N_0 W}$ and $g_{mj}^{r,d} \triangleq \frac{|h_{mj}^{r,d}|^2}{N_0 W}$ characterize the quality of the source-relay and relay-destination wireless channels of this DF relay link. If $g_{mj}^{s,r} \leq g_m^{s,d}$, one can simply show that $R_{mj}^{DF} < R_m^{DT}$, which indicates that DF relaying can not achieve a higher data rate than direct transmission without relay nodes. Therefore, we assume that $g_{mj}^{s,r} > g_m^{s,d}$ is satisfied for all the $J(m)$ relay nodes of data stream m .

Let $s(m)$ represent the source node of data stream m . Then, the power constraint of source node l over all the channels can be determined as

$$\sum_{\{m|s(m)=l\}} \left(P_m^s + \sum_{j \in \mathcal{J}(m)} P_{mj}^s \right) \leq P_{l,\max}^s,$$

where $P_{l,\max}^s$ is the maximal transmission power of source node l . The power constraint of relay

node j is expressed as

$$\sum_{\{m|j \in \mathcal{J}(m)\}} P_{mj}^r \leq P_{j,\max}^r,$$

where $P_{j,\max}^r$ is the maximal transmission power of relay node j . The power allocation problem of the DF relay network is formulated as

$$(P) \max_{P_m^s, P_{mj}^s, P_{mj}^r} \sum_{m=1}^M \left(R_m^{DT} + \sum_{j \in \mathcal{J}(m)} R_{mj}^{DF} \right) \quad (3a)$$

$$\text{s.t.} \quad \sum_{\{m|s(m)=l\}} \left(P_m^s + \sum_{j \in \mathcal{J}(m)} P_{mj}^s \right) \leq P_{l,\max}^s, \forall l \quad (3b)$$

$$\sum_{\{m|j \in \mathcal{J}(m)\}} P_{mj}^r \leq P_{j,\max}^r, \forall j \quad (3c)$$

$$P_m^s, P_{mj}^s, P_{mj}^r \geq 0, \forall m, j. \quad (3d)$$

The achievable rates R_m^{DT} and R_{mj}^{DF} are both concave in their transmission power variables. Therefore, the power allocation problem (P) is a convex optimization problem. However, as we have mentioned earlier, the achievable rate of DF relaying R_{mj}^{DF} is not strictly concave. Specifically, R_{mj}^{DF} is linear in the transmission power variables P_{mj}^s, P_{mj}^r in two cases: When $P_{mj}^s g_{mj}^{s,r} < P_{mj}^s g_m^{s,d} + P_{mj}^r g_{mj}^{r,d}$ holds, the achievable rate R_{mj}^{DF} in (2) does not vary with respect to P_{mj}^r . Moreover, if $P_{mj}^s g_{mj}^{s,r} > P_{mj}^s g_m^{s,d} + P_{mj}^r g_{mj}^{r,d}$ holds and the value of $P_{mj}^s g_m^{s,d} + P_{mj}^r g_{mj}^{r,d}$ is fixed, R_{mj}^{DF} maintains the same value as P_{mj}^r varies.

In dual decomposition based distributed optimization techniques, it is quite difficult to recover the optimal primal variables (i.e. the transmission power variables P_{mj}^s, P_{mj}^r), if the objective function is non-strictly concave [30], [35], [36], [39]. The dual variables can converge to the optimal solution to the dual problem. However, the primal variables may oscillate forever and never result in a feasible solution [38]; see Remark 4 for more details. In the next section, we develop a distributed power allocation algorithm to address this non-strict concavity difficulty, and then prove its convergence to the optimal solution.

III. DISTRIBUTED RESOURCE ALLOCATION ALGORITHM

To circumvent this non-strict concavity difficulty, we use the idea of proximal point method [37], which is to add some quadratic terms and make the objective function strictly concave in

the primal variables. However, a standard proximal point method will not be effective because it relies on a two-layered nested iteration structure. Such a structure is not suitable for distributed implementation, because it is difficult to decide in a distributed manner when the inner-layer iterations have converged. We will overcome this difficulty by developing a single-layer fully distributed algorithm, which converges to the optimal solution. We provide the details as below.

A. Single-layer Distributed Resource Allocation Algorithm

The original problem (P) is modified as the following problem with more variables:

$$\begin{aligned} \max_{\substack{P_m^s, P_{mj}^s, P_{mj}^r \\ Q_m^s, Q_{mj}^s, Q_{mj}^r}} \quad & \sum_{m=1}^M \left[R_m^{DT} - \frac{c_m}{2} (P_m^s - Q_m^s)^2 \right] \\ & + \sum_{m=1}^M \sum_{j \in \mathcal{J}(m)} \left[R_{mj}^{DF} - \frac{c_{mj}}{2} (P_{mj}^s - Q_{mj}^s)^2 \right. \\ & \quad \left. - \frac{c_{mj}}{2} (P_{mj}^r - Q_{mj}^r)^2 \right] \end{aligned} \quad (4a)$$

$$\text{s.t.} \quad \sum_{\{m|s(m)=l\}} \left(P_m^s + \sum_{j \in \mathcal{J}(m)} P_{mj}^s \right) \leq P_{l,\max}^s, \forall l \quad (4b)$$

$$\sum_{\{m|j \in \mathcal{J}(m)\}} P_{mj}^r \leq P_{j,\max}^r, \forall j \quad (4c)$$

$$P_m^s, P_{mj}^s, P_{mj}^r \geq 0, \forall m, j, \quad (4d)$$

where Q_m^s , Q_{mj}^s , and Q_{mj}^r are auxiliary variables corresponding to P_m^s , P_{mj}^s , and P_{mj}^r , respectively. It is easy to show that the optimal value of (4a) coincides with that of (3a) [37]. In fact, let \vec{P}^* denote the maximizer of (P), then $\vec{P} = \vec{P}^*$, $\vec{Q} = \vec{P}^*$ maximize (4). Moreover, problem (4) is strictly concave with respect to the transmission power variables P_m^s , P_{mj}^s , and P_{mj}^r . Small c_m and c_{mj} indicates that the new problem (4) is close to the original problem (P), while large c_m and c_{mj} suggests that the new problem is more strict concave. In the sequent, we solve problem (4) instead of the original problem (P).

Conventionally, the proximal point method requires a two-layer nested optimization structure [37]. The inner-layer of the algorithm optimizes the original primal variables, while the outer-layer updates the introduced auxiliary variables. Each outer-layer update can proceed only after the inner-layer iterations converges, so as to assure the convergence of the algorithm. However, as

mentioned before, such a nested iteration structure is not conducive to distributed implementation. When the network is large, there is no distributed way to know when the inner-layer iterations can stop. To address this difficulty, a modified proximal point algorithm with a single-layer iteration structure was proposed in [38] for the multi-path routing problem. However, the arguments of [38] are quite specific to the structure of the objective function, and do not apply to our problem (4).

Let μ_l and ν_j be the Lagrange multipliers associated with the constraints in (4b) and (4c), respectively. The partial Lagrangian of problem (4) with respect to the power constraints (4b) and (4c) is given by

$$\begin{aligned}
& L(P_m^s, P_{mj}^s, P_{mj}^r, Q_m^s, Q_{mj}^s, Q_{mj}^r; \mu_l, \nu_j) \\
&= \sum_{m=1}^M \left\{ \left[R_m^{DT} - \frac{c_m}{2} (P_m^s - Q_m^s)^2 \right] \right. \\
&\quad + \sum_{j \in \mathcal{J}(m)} \left[R_{mj}^{DF} - \frac{c_{mj}}{2} (P_{mj}^s - Q_{mj}^s)^2 \right. \\
&\quad \quad \left. \left. - \frac{c_{mj}}{2} (P_{mj}^r - Q_{mj}^r)^2 \right] \right\} \\
&\quad - \sum_{l=1}^N \mu_l \left[\sum_{\{m|s(m)=l\}} \left(P_m^s + \sum_{j \in \mathcal{J}(m)} P_{mj}^s - P_{l,\max}^s \right) \right] \\
&\quad - \sum_{j=1}^J \nu_j \left(\sum_{\{m|j \in \mathcal{J}(m)\}} P_{mj}^r - P_{j,\max}^r \right). \tag{5}
\end{aligned}$$

For convenience, we rearrange the above Lagrangian as

$$\begin{aligned}
& L(\vec{P}, \vec{Q}; \vec{\nu}) \\
&= R(\vec{P}) - \frac{1}{2} (\vec{P} - \vec{Q})^T V (\vec{P} - \vec{Q}) - \vec{\nu}^T (E\vec{P} - \vec{P}_{\max}), \tag{6}
\end{aligned}$$

where $R(\vec{P})$ is the objective function of problem (P), \vec{P} is a $M + 2 \sum_{m=1}^M J(m)$ dimensional vector representing the power allocation variables P_m^s, P_{mj}^s , and P_{mj}^r , \vec{Q} is a $M + 2 \sum_{m=1}^M J(m)$ dimensional vector representing the auxiliary variables Q_m^s, Q_{mj}^s , and Q_{mj}^r , \vec{P}_{\max} is a $N + J$ dimensional vector representing the maximal transmission power $P_{l,\max}^s$ and $P_{j,\max}^r$, $\vec{\nu}$ is a $N + J$ dimensional vector representing the dual variables μ_l and ν_j , and E is $(N + J) \times (M + 2 \sum_{m=1}^M J(m))$ dimensional vector representing the relationship between the transmitting power variables and corresponding source/relay nodes.

Our proposed distributed power allocation algorithm is described as follows:

Algorithm A: In the k th iteration,

(A1) Suppose $\vec{x}(k)$ is the optimal power allocation solution for fixed $\vec{\nu}(k)$, $\vec{Q}(k)$:

$$\vec{x}(k) = \arg \max_{\vec{P} \geq 0} L \left(\vec{P}, \vec{Q}(k); \vec{\nu}(k) \right). \quad (7)$$

Update the dual variables $\vec{\nu}(k+1)$ as

$$\vec{\nu}(k+1) = \left\{ \vec{\nu}(k) + A \left[E\vec{x}(k) - \vec{P}_{\max} \right] \right\}^+, \quad (8)$$

where A is a $(N+J) \times (N+J)$ dimensional diagonal matrix with diagonal elements α_l ($l = 1, 2, \dots, N+J$) as the step-size of dual updates, and $(\cdot)^+ \triangleq \max\{\cdot, 0\}$.

(A2) Update the auxiliary variable $\vec{Q}(k+1)$ as

$$\vec{Q}(k+1) = \arg \max_{\vec{P} \geq 0} L \left(\vec{P}, \vec{Q}(k); \vec{\nu}(k+1) \right). \quad (9)$$

Remark 1: A straightforward method to solve (4) is the two-layer nested proximal point method [37], which can be expressed as follows: In the inner layer iterations, the transmission power variable \vec{P} is optimized for fixed \vec{Q} by Lagrangian dual optimization method, resulting in an optimization problem of \vec{Q} . After the inner-layer iterations have converged, the auxiliary variable \vec{Q} is updated in the outer layer. Such a structure is not suitable for distributed implementation, because it is difficult to decide in a distributed manner when the inner-layer iterations have converged. On the other hand, the proposed Algorithm A has a nice single-layer optimization structure. The outer-layer update of \vec{Q} does not request that the inner-layer dual updates have converged.

Remark 2: If the traditional proximal point method is used to solve (4), the convergence analysis will not involve the dual variable ν [37], because the inner-layer dual optimization has converged. However, in order to show the convergence of Algorithm A, we need to characterize the influence of the dual update (8). Therefore, it is more difficult to establish the convergence of Algorithm A.

B. Distributed Implementation of Algorithm A

We proceed to explain how to implement Algorithm A in a distributed fashion. In particular, we show that each step of Algorithm A only needs local information exchange among the source, relay, and destination nodes of each DF relay link.

First, the dual update (8) can be equivalently expressed as

$$\mu_l(k+1) = \left[\mu_l(k) + a_l \left(P_m^s + \sum_{j \in \mathcal{J}(m)} P_{mj}^s - P_{l,\max}^s \right) \right]^+, \quad (10)$$

$$\nu_j(k+1) = \left[\nu_j(k) + a_{N+j} \left(\sum_{j \in \mathcal{J}(m)} P_{mj}^r - P_{j,\max}^r \right) \right]^+, \quad (11)$$

which can be carried out distributedly at each source and relay nodes. Moreover, the Lagrangian maximization problems in (7) and (9) can be decomposed into many independent local power allocation subproblems. Specifically, the terms of the Lagrangian L in (5) can be reassembled as

$$\begin{aligned} & L(\vec{P}, \vec{Q}; \vec{\nu}) \\ &= \sum_{m=1}^M \left\{ \left[R_m^{DT} - \frac{c_m}{2} (P_m^s - Q_m^s)^2 - \mu_{s(m)} P_m^s \right] \right. \\ &\quad + \sum_{j \in \mathcal{J}(m)} \left[R_{mj}^{DF} - \frac{c_{mj}}{2} (P_{mj}^s - Q_{mj}^s)^2 - \frac{c_{mj}}{2} (P_{mj}^r - Q_{mj}^r)^2 \right. \\ &\quad \left. \left. - \mu_{s(m)} P_{mj}^s - \nu_j P_{mj}^r \right] \right\} + \sum_{l=1}^N \mu_l P_{l,\max}^s + \sum_{j=1}^J \nu_j P_{j,\max}^r. \end{aligned} \quad (12)$$

Therefore, the Lagrangian maximization problem in (7) and (9) can be rewritten as

$$\begin{aligned} & \max_{\vec{P} \geq 0} L(\vec{P}, \vec{Q}; \vec{\nu}) \\ &= \sum_{m=1}^M \left[H_m(Q_m^s; \mu_{s(m)}) + \sum_{j \in \mathcal{J}(m)} I_{mj}(Q_{mj}^s, Q_{mj}^r; \mu_{s(m)}, \nu_j) \right] \\ &\quad + \sum_{l=1}^N \mu_l P_{l,\max}^s + \sum_{j=1}^J \nu_j P_{j,\max}^r, \end{aligned} \quad (13)$$

where

$$\begin{aligned} & H_m(Q_m^s; \mu_{s(m)}) \\ &= \max_{P_m^s \geq 0} R_m^{DT} - \frac{c_m}{2} (P_m^s - Q_m^s)^2 - \mu_{s(m)} P_m^s, \end{aligned} \quad (14)$$

$$\begin{aligned} & I_{mj}(Q_{mj}^s, Q_{mj}^r; \mu_{s(m)}, \nu_j) \\ &= \max_{P_{mj}^s, P_{mj}^r \geq 0} R_{mj}^{DF} - \frac{c_{mj}}{2} (P_{mj}^s - Q_{mj}^s)^2 \\ & \quad - \frac{c_{mj}}{2} (P_{mj}^r - Q_{mj}^r)^2 - \mu_{s(m)} P_{mj}^s - \nu_j P_{mj}^r, \end{aligned} \quad (15)$$

are local power allocation subproblems for the DT link and DF relay link, respectively. The closed-form solutions to (14) and (15) are provided in the following lemmas, where the subscripts are omitted for ease of notation:

Lemma 1: The optimal solution to (14) is

$$P^s = f(2\theta^{DT}, c, \mu, Q^s, g^{s,d}, 1), \quad (16)$$

where

$$f(\theta, c, \mu, Q, g, v) \triangleq \frac{1}{2} \left(\frac{\theta}{\mu \ln 2} - \frac{\theta}{g} + \sqrt{x^2 + y} - x \right)^+, \quad (17)$$

$$x = \frac{\mu}{cv} - \frac{Q}{v} + \frac{\theta}{\mu \ln 2} - \frac{\theta}{2g}, \quad (18)$$

$$y = \frac{2Q\theta}{\mu v \ln 2} + \frac{\theta^2}{g\mu \ln 2} - \frac{\theta^2}{\mu^2 \ln 2^2}. \quad (19)$$

The proof of Lemma 1 is provided in Appendix A. As $c \rightarrow 0$, $\sqrt{x^2 + y} - x$ also tends to 0. In this case, (16) reduces to the conventional water-filling solution.

Lemma 2: The optimal solution to (15) is provided for three separate cases:

Case 1: if $g^{r,d}P^r > (g^{s,r} - g^{s,d})P^s$, the optimal values of P^s and P^r are given by

$$\begin{cases} P^s = f(\theta^{DF}, c, \mu, Q^s, g^{s,r}, 1), \\ P^r = [-\nu/c + Q^r]^+. \end{cases} \quad (20)$$

Case 2: $g^{r,d}P^r < (g^{s,r} - g^{s,d})P^s$. If P^r in (21) satisfies $P^r \geq 0$, the optimal values of P^s and

P^r are given by

$$\begin{cases} P^s = \frac{g^{r,d}(g^{r,d}\nu - g^{s,d}\mu)}{[(g^{s,d})^2 + (g^{r,d})^2]c} + \frac{g^{r,d}(g^{s,d}Q^s - g^{r,d}Q^r)}{(g^{s,d})^2 + (g^{r,d})^2} \\ \quad + \frac{g^{s,d}e}{(g^{s,d})^2 + (g^{r,d})^2}, \\ P^r = -\frac{g^{s,d}(g^{r,d}\nu - g^{s,d}\mu)}{[(g^{s,d})^2 + (g^{r,d})^2]c} - \frac{g^{s,d}(g^{s,d}Q^s - g^{r,d}Q^r)}{(g^{s,d})^2 + (g^{r,d})^2} \\ \quad + \frac{g^{r,d}e}{(g^{s,d})^2 + (g^{r,d})^2}, \end{cases} \quad (21)$$

where e is the value of $P^s g^{s,d} + P^r g^{r,d}$ given by

$$\begin{aligned} e = f \left(\theta^{DF} \left[(g^{s,d})^2 + (g^{r,d})^2 \right], c, g^{s,d}\mu + g^{r,d}\nu, \right. \\ \left. g^{s,d}Q^s + g^{r,d}Q^r, (g^{s,d})^2 + (g^{r,d})^2, 1 \right), \end{aligned} \quad (22)$$

Otherwise, if P^r in (21) is negative, the optimal values of P^s and P^r are given by

$$\begin{cases} P^s = f(\theta^{DF}, c, \mu, Q^s, g^{s,d}, 1), \\ P^r = 0. \end{cases} \quad (23)$$

Case 3: if $g^{r,d}P^r = (g^{s,r} - g^{s,d})P^s$, the optimal values of P^s and P^r are given by

$$\begin{cases} P^s = f \left(\theta^{DF}, c, \mu + \frac{\nu(g^{s,r} - g^{s,d})}{g^{r,d}}, \right. \\ \quad \left. Q^s + \frac{Q^r(g^{s,r} - g^{s,d})}{g^{r,d}}, g^{s,r}, 1 + \frac{(g^{s,r} - g^{s,d})^2}{(g^{r,d})^2} \right), \\ P^r = \frac{P^s(g^{s,r} - g^{s,d})}{g^{r,d}}. \end{cases} \quad (24)$$

The proof of Lemma 2 is provided in Appendix B.

Remark 3: The solution to (14) requires only the CSI $g^{s,d}$ and the latest dual variable μ of the source node. The solution to (15) requires only the CSI $g^{s,r}$, $g^{s,d}$, and $g^{r,d}$, as well as the latest dual variables μ and ν of source and relay nodes. Therefore, Algorithm \mathcal{A} only requires local information exchange among the source, relay, and destination nodes of each DF relay link.

Remark 4: Subproblem (15) has unique solution for all the values of the dual variables μ and ν , thanks to the introduced quadratic terms. On the other hand, if $c = 0$, problem (4) reduces to the original problem (P), and the subproblem (15) may have many optimal solutions. First, if $\nu = c = 0$, ν/c in the expression of P^r in (20) can be an arbitrary positive number in $[0, Q^r]$. Second, the source and relay power allocation in (21) satisfies (see also (56) in Appendix A)

$$[\mu + c(P^s - Q^s)]g^{r,d} - [\nu + c(P^r - Q^r)]g^{s,d} = 0. \quad (25)$$

If $c = 0$, we have $g^{r,d}\mu - g^{s,d}\nu = 0$ from (25). Therefore, both the numerator and denominator in the first terms of source/relay power in (21) are zero, and the optimal source and relay power solutions are non-unique. Since no global network information is available when solving the local power allocation subproblem (15), it is quite difficult to find a global feasible solution among all the local optimal solutions. As a result, the power allocation variable will keep oscillating, although the dual variable converges to the optimal solution.

C. Convergence Analysis of Algorithm \mathcal{A}

Let us define the stationary point of Algorithm \mathcal{A} :

Definition 1: A point $(\vec{Q}^*, \vec{\nu}^*)$ is a *stationary point* of Algorithm \mathcal{A} , if

$$\vec{Q}^* = \arg \max_{\vec{x} \geq 0} L(\vec{x}, \vec{Q}^*; \vec{\nu}^*), \quad (26)$$

$$E\vec{Q}^* - \vec{P}_{\max} \leq 0, \quad \vec{\nu}^* \geq 0, \quad (27)$$

$$\vec{\nu}^* \otimes (E\vec{Q}^* - \vec{P}_{\max}) = 0, \quad (28)$$

where $x \otimes y$ represents the hadamard (elementwise) product of two vectors x and y with the same dimension.

Since problem (P) only has affine power constraints, it satisfies the modified Slater's condition [40], if it is feasible. The following result holds for the stationary point of Algorithm \mathcal{A} :

Lemma 3: If problem (P) is feasible, $(\vec{Q}^*, \vec{\nu}^*)$ is a stationary point of Algorithm \mathcal{A} , then \vec{Q}^* is one optimal solution to problem (P).

Proof: Suppose that

$$\vec{P} = \arg \max_{\vec{x} \geq 0} L(\vec{x}, \vec{Q}; \vec{\nu}). \quad (29)$$

According to the Karush-Kuhn-Tucker (KKT) condition [40], there must exist a subgradient $\nabla R(\vec{P})$ of $R(\vec{P})$ such that

$$\nabla R(\vec{P}) - E^T \vec{\nu} - V(\vec{P} - \vec{Q}) = 0. \quad (30)$$

By comparing (26) with (29), the following equation must hold for the stationary point $(\vec{Q}^*, \vec{\nu}^*)$:

$$\nabla R(\vec{Q}^*) - E^T \vec{\nu}^* = 0. \quad (31)$$

Assembling (27), (28), and (31), the stationary point $(\vec{Q}^*, \vec{\nu}^*)$ satisfies the KKT optimality conditions of problem (P). According to the modified Slater's condition, \vec{Q}^* is one optimal solution to problem (P) [40]. ■

By the concavity of $R(\vec{P})$, we have

$$\left[\nabla R(\vec{P}) - \nabla R(\vec{Q}^*) \right]^T (\vec{P} - \vec{Q}^*) \leq 0. \quad (32)$$

The following key result can be viewed as an extension of the above inequality.

Lemma 4: Let $(\vec{P}_1, \vec{\nu}_1)$ and $(\vec{P}_2, \vec{\nu}_2)$ be the corresponding maximizers of the Lagrangian (6) for fixed \vec{Q} , i.e. $\vec{P}_1 = \arg \max_{\vec{P} \geq 0} L(\vec{P}, \vec{Q}, \vec{\theta}; \vec{\nu}_1)$ and $\vec{P}_2 = \arg \max_{\vec{P} \geq 0} L(\vec{P}, \vec{Q}, \vec{\theta}; \vec{\nu}_2)$. If c_{mj} is small enough, then

$$\begin{aligned} & \left[\nabla R(\vec{P}_1) - \nabla R(\vec{Q}^*) \right]^T (\vec{P}_2 - \vec{Q}^*) \\ & \leq \frac{1}{2} (\vec{\nu}_2 - \vec{\nu}_1)^T EV^{-1}E (\vec{\nu}_2 - \vec{\nu}_1), \end{aligned} \quad (33)$$

where $\nabla R(\vec{P}_1)$ and $\nabla R(\vec{Q}^*)$ are defined in (30) and (31), respectively.

Proof: See Appendix C. ■

As we mentioned in Remark 2, a key step in establishing the convergence of Algorithm \mathcal{A} is to characterize the influence of the dual update (8). In Lemma 4, we established a relationship between two Lagrangian maximizers $(\vec{P}_1, \vec{\nu}_1)$ and $(\vec{P}_2, \vec{\nu}_2)$. Suppose that the dual variable is updated from $\vec{\nu}_2$ to $\vec{\nu}_1$, if the step-size of the dual update between $\vec{\nu}_1$ and $\vec{\nu}_2$ is not large, the left hand side of (33) will not be far above zero.

A similar result to Lemma 4 was obtained in [38], however we cannot use the arguments in [38] to prove Lemma 4. In particular, the arguments in [38] require the following relationship

$$\left[\nabla_{P_{mj}^s} R(\vec{P}_1) - \nabla_{P_{mj}^s} R(\vec{Q}^*) \right] (P_{mj,1}^s - Q_{mj}^{s*}) \leq 0 \quad (34)$$

and

$$\left[\nabla_{P_{mj}^r} R(\vec{P}_1) - \nabla_{P_{mj}^r} R(\vec{Q}^*) \right] (P_{mj,1}^r - Q_{mj}^{r*}) \leq 0. \quad (35)$$

Since $R(\vec{P})$ is *jointly concave* with respect to P_{mj}^s and P_{mj}^r , instead of (34) and (35), we only have

$$\left[\nabla_{P_{mj}^s} R(\vec{P}_1) - \nabla_{P_{mj}^s} R(\vec{Q}^*) \right] (P_{mj,1}^s - Q_{mj}^{s*}) \quad (36)$$

$$+ \left[\nabla_{P_{mj}^r} R(\vec{P}_1) - \nabla_{P_{mj}^r} R(\vec{Q}^*) \right] (P_{mj,1}^r - Q_{mj}^{r*}) \leq 0. \quad (37)$$

Therefore, new techniques are developed to show Lemma 4 in Appendix C.

Let us define the norm of dual and auxiliary variables:

$$\|\vec{v}\|_A \triangleq \sqrt{\vec{v}^T A^{-1} \vec{v}}, \quad \|\vec{Q}\|_V \triangleq \sqrt{\vec{Q}^T V \vec{Q}}. \quad (38)$$

With Lemma 4, we can use the Lyapunov drift techniques to prove the following theorem as in [38]:

Theorem 1: If the algorithm parameters c_m , c_{mj} is small enough, and the dual step-size α_n satisfies

$$\max_l \alpha_l \leq \frac{1}{2S} \min_{m,j} \{c_m, c_{mj}\}, \quad (39)$$

our proposed Algorithm \mathcal{A} converges to a stationary point, i.e., one optimal solution to problem (P), where S is the maximal number of links that a source or relay node can participate, given by

$$S = \max \left\{ \max_l \left[\sum_{\{m|s(m)=l\}} (1 + J(m)) \right], \max_j \left[\sum_{\{m|j \in \mathcal{J}(m)\}} 1 \right] \right\}. \quad (40)$$

The proof of Theorem 1 is omitted here, because once Lemma 4 is established, Theorem 1 follows from combining it with standard Lyapunov drift techniques. For the readers' convenience, we provide details in our online technical report [41].

IV. NUMERICAL RESULTS

This section provides some numerical results of our proposed distributed power allocation Algorithm \mathcal{A} . A DF relay network with 4 user (source/destination) nodes and 2 relay nodes are considered, with a network topology illustrated in Fig. 3. The channel power gain between two nodes is determined by a large-scale path loss component with path loss factor of 4. We assume that each source and relay node has the same amount of transmission power, i.e. $P_{l,\max}^s = P_{j,\max}^r = P_{\max}$ for all l, j , and the received signal-to-noise ratio (SNR) at unit distance from a transmitting node is $\frac{P_{\max}}{N_0 W} = 20\text{dB}$. There are 5 data streams in this network (see Fig. 3), that share the total time/frequency channel resource equally. If DF relay techniques are not employed, the channel resource proportion of each DT link is $\theta_m^{DT} = \frac{1}{5}$. When both DT and DF relay transmissions are allowed, data stream 1 can only make use of relay 1, but not relay 2,

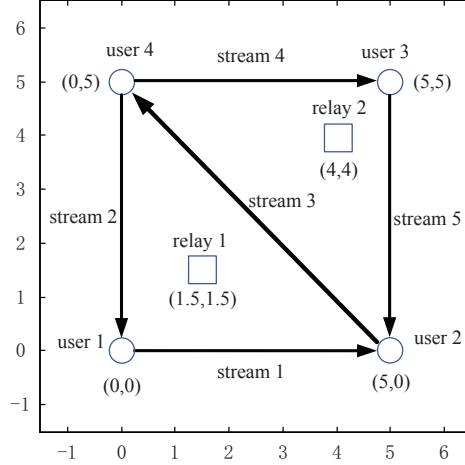


Fig. 3. The topology of considered DF relay network.

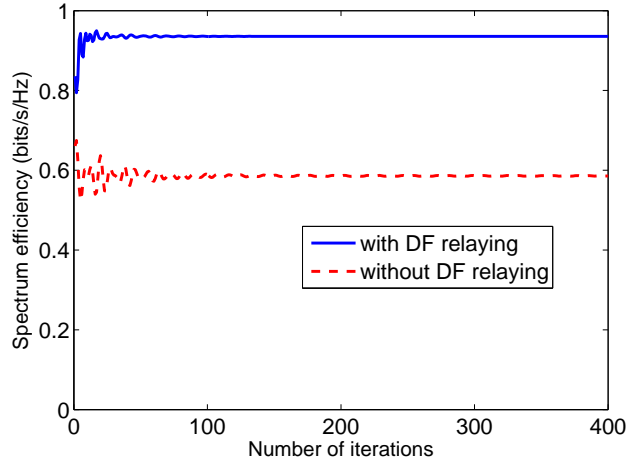


Fig. 4. Convergence performance of our distributed power allocation Algorithm \mathcal{A} .

since $g_{12}^{s,r} < g_{12}^{s,d}$. The other data streams can be assisted by both relay 1 and 2. The channel resource of each data stream is equally allocated to its DT and DF relay links.

Figure 4 illustrates the convergence performance of our distributed power allocation Algorithm \mathcal{A} . The algorithm parameters are $c_m = c_{mj} = 10^{-4}$, $\alpha_l = 5 \times 10^{-5}$. The achievable rate of each data stream is shown in Tab. I for both the cases with and without DF relay techniques. The spectrum efficiency of each data stream increases significantly by deploying the DF relay nodes, and the total network spectrum efficiency increases 59.7%.

Figure 5 provides the evolution of power allocation variables for the DT and DF links of data

TABLE I
SPECTRUM EFFICIENCY OF EACH DATA STREAM (BITS/S/Hz)

	Stream 1	Stream 2	Stream 3	Stream 4	Stream 5
w/ relay	0.2700	0.1347	0.1702	0.1293	0.2313
w/o relay	0.1696	0.0971	0.0523	0.0971	0.1696

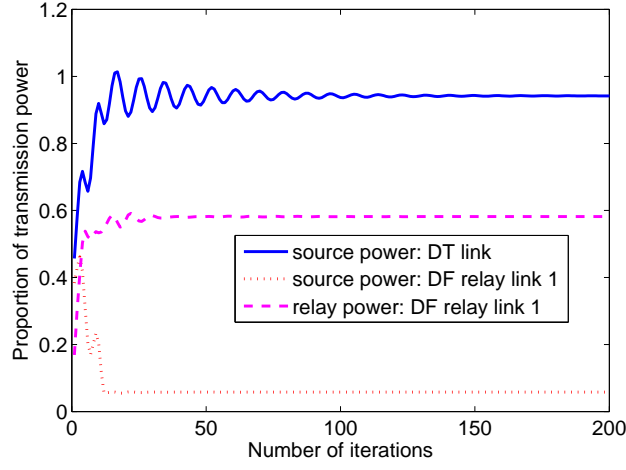


Fig. 5. Evolution of power allocation variables for the DT and DF links of data stream 1.

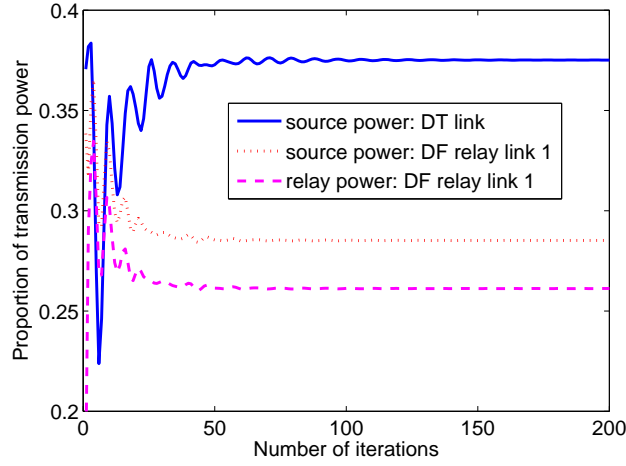


Fig. 6. Evolution of power allocation variables for the DT link and one DF link of data stream 3.

stream 1. Since user 1 (the source node of stream 1) is quite near to relay 1, user 1 only needs to spare a small proportion of source power to forward information to relay 1, and yet achieves a

high data rate improvement. Figure 6 further provides the evolution of power allocation variables for the DT link and one DF link of data stream 3. Since user 2 (the source node of stream 3) is relative far from relay 1, user 2 needs to utilize more transmission power in its DF relay link, so as to achieve a high data rate.

V. CONCLUSION

In this paper, we have developed a distributed power allocation algorithm to maximize the network throughput of DF relay networks. This problem is a convex optimization problem, where the objective function is not strictly concave. However, standard proximal techniques are not suitable because they resulted in a nested solution that is difficult to implement. We overcome this difficulty by developing an algorithm whose key feature is a single-layer iteration structure, which is desirable for on-line implementation. In each iteration, information exchange only occurs among the source, relay, and destination nodes of each DF relay link. We show that the algorithm converges to the optimal solution and study its efficacy via numerical evaluation.

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APPENDIX

A. Proof of Lemma 1

The Karush-Kuhn-Tucker (KKT) conditions [40] of (14) imply

$$\frac{g^{s,d}}{\ln 2(1 + \frac{g^{s,d}P^s}{\theta^{DT}})} - c(P^s - Q^s) - \mu \begin{cases} = 0, & \text{if } P^s > 0 \\ \leq 0, & \text{if } P^s = 0 \end{cases}. \quad (41)$$

When $P^s > 0$, (41) achieves equality, P^s is thus the positive root of a quadratic equation equivalent with (41); otherwise, $P^s = 0$. Summarizing these two cases, the value of P^s is given by (16).

B. Proof of Lemma 2

Let us define the rate functions

$$R_1 = \frac{\theta^{DF}}{2} \log_2 \left[1 + \frac{2(P^s g^{s,d} + P^r g^{r,d})}{\theta^{DF}} \right], \quad (42)$$

$$R_2 = \frac{\theta^{DF}}{2} \log_2 \left(1 + \frac{2P^s g^{s,r}}{\theta^{DF}} \right). \quad (43)$$

Therefore, the achievable rate of DF relaying can be expressed as $R^{DF} = \min \{R_1, R_2\}$. Problem (15) is equivalent to the following problem:

$$\max_{t, P^s, P^r \geq 0} t - \frac{c}{2} (P^s - Q^s)^2 - \frac{c}{2} (P^r - Q^r)^2 - \mu P^s - \nu P^r \quad (44a)$$

$$\text{s.t. } t \leq R_1, \quad t \leq R_2, \quad (44b)$$

where R_1 and R_2 are defined in (42) and (43). The Lagrangian of problem (44) is

$$\begin{aligned} L(t, P^s, P^r; \tau, \zeta, \mu, \nu) \\ = t + \tau(R_1 - t) + \zeta(R_2 - t) - \frac{c}{2} (P^s - Q^s)^2 \\ - \frac{c}{2} (P^r - Q^r)^2 - \mu P^s - \nu P^r. \end{aligned} \quad (45)$$

The KKT optimality conditions of problem (44) indicate

$$\frac{\partial L}{\partial t} = 1 - \tau - \zeta = 0, \quad (46)$$

$$\tau \geq 0, \quad R_1 - t \geq 0, \quad \tau(R_1 - t) = 0, \quad (47)$$

$$\zeta \geq 0, \quad R_2 - t \geq 0, \quad \zeta(R_2 - t) = 0. \quad (48)$$

By (46), the Lagrangian (45) can be simplified as

$$\begin{aligned} L(P^s, P^r; \tau, \mu, \nu) \\ = \tau R_1 + (1 - \tau) R_2 - \frac{c}{2} (P^s - Q^s)^2 - \frac{c}{2} (P^r - Q^r)^2 \\ - \mu P^s - \nu P^r. \end{aligned} \quad (49)$$

Moreover, from the KKT optimality conditions (46)-(48), we obtain

$$\begin{cases} \text{if } R_1^* > R_2^*, & \tau = 0; \\ \text{if } R_1^* < R_2^*, & \tau = 1; \\ \text{if } R_1^* = R_2^*, & 0 \leq \tau \leq 1, \end{cases} \quad (50)$$

where R_1^* and R_2^* are corresponding rate values (42) and (43) at the optimal power allocation solution. Therefore, problem (44) can be address by solving the Lagrangian maximization problem

$$\max_{P^s, P^r \geq 0} L(P^s, P^r; \tau, \mu, \nu), \quad (51)$$

for the 3 cases expressed in (50).

Case 1: When $R_1^* > R_2^*$, we have $\tau = 0$. The KKT conditions of (51) are

$$\frac{g^{s,r}}{\ln 2(1 + \frac{2g^{s,r}P^s}{\theta^{DF}})} - \mu + c(P^s - Q^s) \begin{cases} = 0, \text{ if } P^s > 0 \\ \leq 0, \text{ if } P^s = 0 \end{cases}, \quad (52)$$

$$- \nu - c(P^r - Q^r) \begin{cases} = 0, \text{ if } P^r > 0 \\ \leq 0, \text{ if } P^r = 0 \end{cases}. \quad (53)$$

The solution to (52) is similar to that of (41). Further considering (53), the optimal source and relay power is given by (20). Note that Case 1 requires $R_1^* > R_2^*$, which can be equivalently expressed by $g^{r,d}P^r > (g^{s,r} - g^{s,d})P^s$.

Case 2: When $R_1^* < R_2^*$, $\tau = 1$. The KKT conditions of (51) are given by

$$\frac{g^{s,d}}{\ln 2(1 + \frac{2g^{s,d}P^s + 2g^{r,d}P^r}{\theta^{DF}})} - \mu - c(P^s - Q^s) \begin{cases} = 0, \text{ if } P^s > 0 \\ \leq 0, \text{ if } P^s = 0 \end{cases}, \quad (54)$$

$$\frac{g^{r,d}}{\ln 2(1 + \frac{2g^{s,d}P^s + 2g^{r,d}P^r}{\theta^{DF}})} - \nu - c(P^r - Q^r) \begin{cases} = 0, \text{ if } P^r > 0 \\ \leq 0, \text{ if } P^r = 0 \end{cases}. \quad (55)$$

If $P^s > 0, P^r > 0$, then (54) and (55) take equality. By viewing $e = g^{s,d}P^s + g^{r,d}P^r$ as a whole body, we can get a quadratic equation of e from (54) and (55), which has a positive root given by (22). Moreover, by comparing (54) and (55) with equality, we obtain

$$[\mu + c(P^s - Q^s)] g^{r,d} = [\nu + c(P^r - Q^r)] g^{s,d}. \quad (56)$$

Substituting (56) into (22), the optimal power allocation solution is derived as in (21).

If $P^r = 0$, (54) reduces to a formula similar with (52), and its solution is given by (23).

Note that Case 2 requires $R_1^* < R_2^*$, which is guaranteed by $g^{r,d}P^r < (g^{s,r} - g^{s,d})P^s$. The case of $P^s = 0$ and $P^r > 0$ can not happen, since it violates the condition $R_1^* < R_2^*$.

Case 3: If $R_1^* = R_2^*$, the KKT conditions of (51) are given by

$$\begin{aligned} & \frac{\tau g^{s,d}}{\ln 2(1 + \frac{2g^{s,d}P^s + 2g^{r,d}P^r}{\theta^{DF}})} + \frac{(1 - \tau)g^{s,r}}{\ln 2(1 + \frac{2g^{s,r}P^s}{\theta^{DF}})} \\ & - \mu - c(P^s - Q^s) \begin{cases} = 0, & \text{if } P^s > 0 \\ \leq 0, & \text{if } P^s = 0 \end{cases}, \end{aligned} \quad (57)$$

$$\begin{aligned} & \frac{\tau g^{r,d}}{\ln 2(1 + \frac{2g^{s,d}P^s + 2g^{r,d}P^r}{\theta^{DF}})} \\ & - \nu - c(P^r - Q^r) \begin{cases} = 0, & \text{if } P^r > 0 \\ \leq 0, & \text{if } P^r = 0 \end{cases}. \end{aligned} \quad (58)$$

Since $R_1^* = R_2^*$, we have

$$g^{r,d}P^r = (g^{s,r} - g^{s,d})P^s. \quad (59)$$

If $P^s > 0, P^r > 0$, both (57) and (58) achieves equality. By substituting (59) into (57) and (58), we can eliminate τ and derive the optimal values of P^s and P^r . Otherwise, $P^s = P^r = 0$. These two cases are summarized in (24).

C. Proof of Lemma 4

We proceed to show the inequalities

$$\begin{aligned} & \left[\nabla_{P_m^s} R_m^{DT}(P_{m,1}^s) - \nabla_{P_m^s} R_m^{DT}(Q_m^{s*}) \right]^T (P_{m,2}^s - Q_m^{s*}) \\ & \leq \frac{1}{2c_m} (\mu_{s(m),2} - \mu_{s(m),1})^2, \forall m, \end{aligned} \quad (60)$$

and

$$\begin{aligned} & \left[\nabla_{\vec{P}_{mj}} R_{mj}^{DF}(\vec{P}_{mj,1}) - \nabla_{\vec{P}_{mj}} R_{mj}^{DF}(\vec{Q}_{mj}^*) \right]^T (\vec{P}_{mj,2} - \vec{Q}_{mj}^*) \\ & \leq \frac{1}{2c_{mj}} [(\mu_{s(m),2} - \mu_{s(m),1})^2 + (\nu_{j,2} - \nu_{j,1})^2], \forall m, j, \end{aligned} \quad (61)$$

where $P_{m,i}^s$ is the maximizer of (14) corresponding to the multiplier $\mu_{s(m),i}$ for $i \in \{1, 2\}$, and $\vec{P}_{mj,i} = (P_{mj,i}^s, P_{mj,i}^r)$ is the maximizer of (15) corresponding the multiplier $(\mu_{s(m),i}, \nu_{j,i})$. The asserted result (33) follows, if we take the summation of the inequalities (60) and (61) for all

the possible choices of m and j . Since R_m^{DT} is a concave function of a single variable P_m^s , the techniques of [38] can be directly used here to prove (60). In the sequel, we will show (61) for each DF relay link. Since we only need to focus on one DF relay link, the subscripts $m, j, s(m)$ are omitted in the sequel to facilitate our expressions.

Let us associate Lagrange multipliers $L^s \geq 0$ and $L^r \geq 0$ for the constraints $P^s \geq 0$ and $P^r \geq 0$, respectively, in the maximization of (15). Using the Karush-Kuhn-Tucker condition, we can conclude that there must exist a subgradient $(\frac{\partial R^{DF}(\vec{P})}{\partial P^s}, \frac{\partial R^{DF}(\vec{P})}{\partial P^r})$ of R^{DF} such that

$$\frac{\partial R^{DF}(\vec{P})}{\partial P^s} - \mu - c(P^s - Q^s) + L^s = 0, \quad (62)$$

$$\frac{\partial R^{DF}(\vec{P})}{\partial P^r} - \nu - c(P^r - Q^r) + L^r = 0, \quad (63)$$

$$L^s P^s = 0, \quad L^r P^r = 0, \quad (64)$$

where $\vec{P} = (P^s, P^r)$ represents the source and relay power of the considered DF relay link. From (30) and (31), we also have

$$\nabla_{P^s} R^{DF}(\vec{P}) - \mu - c(P^s - Q^s) = 0, \quad (65)$$

$$\nabla_{P^r} R^{DF}(\vec{P}) - \nu - c(P^r - Q^r) = 0. \quad (66)$$

Comparing (62) and (63) with (65) and (66), we see that

$$\begin{aligned} \nabla_{P^s} R^{DF}(\vec{P}) &= \frac{\partial R^{DF}(\vec{P})}{\partial P^s} + L^s, \\ \nabla_{P^r} R^{DF}(\vec{P}) &= \frac{\partial R^{DF}(\vec{P})}{\partial P^r} + L^r. \end{aligned}$$

Let $\vec{Q}^* = (Q^{s*}, Q^{r*})$ be the the source and relay power at the stationary point. Similarly, we can obtain

$$\begin{aligned} \nabla_{P^s} R^{DF}(\vec{Q}^*) &= \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^s} + L^{s*}, \\ \nabla_{P^r} R^{DF}(\vec{Q}^*) &= \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^r} + L^{r*}. \end{aligned}$$

Then, we further have

$$\begin{aligned}
& \left[\nabla R^{DF}(\vec{P}_1) - \nabla R^{DF}(\vec{Q}^*) \right]^T (\vec{P}_2 - \vec{Q}^*) \\
&= \left[\frac{\partial R^{DF}(\vec{P}_1)}{\partial P^s} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^s} \right] (P_2^s - Q^{s*}) \\
&+ \left[\frac{\partial R^{DF}(\vec{P}_1)}{\partial P^r} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^r} \right] (P_2^r - Q^{r*}) \\
&+ (L_1^s - L^{s*})(P_2^s - Q^{s*}) + (L_1^r - L^{r*})(P_2^r - Q^{r*}).
\end{aligned}$$

We can use the arguments in [38] to show that

$$\begin{aligned}
& (L_1^s - L^{s*})(P_2^s - Q^{s*}) + (L_1^r - L^{r*})(P_2^r - Q^{r*}) \\
&\leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2].
\end{aligned}$$

Now we only need to show

$$\begin{aligned}
& \left[\frac{\partial R^{DF}(\vec{P}_1)}{\partial P^s} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^s} \right] (P_2^s - Q^{s*}) \\
&+ \left[\frac{\partial R^{DF}(\vec{P}_1)}{\partial P^r} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^r} \right] (P_2^r - Q^{r*}) \\
&\leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2].
\end{aligned} \tag{67}$$

for (61) to hold.

Let us further define

$$\begin{aligned}
a_1^s &= \frac{\partial R^{DF}(\vec{P}_1)}{\partial P^s} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^s}, \quad b_1^s = P_1^s - Q^{s*}, \\
a_1^r &= \frac{\partial R^{DF}(\vec{P}_1)}{\partial P^r} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^r}, \quad b_1^r = P_1^r - Q^{r*}, \\
a_2^s &= \frac{\partial R^{DF}(\vec{P}_2)}{\partial P^s} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^s}, \quad b_2^s = P_2^s - Q^{s*}, \\
a_2^r &= \frac{\partial R^{DF}(\vec{P}_2)}{\partial P^r} - \frac{\partial R^{DF}(\vec{Q}^*)}{\partial P^r}, \quad b_2^r = P_2^r - Q^{r*}.
\end{aligned}$$

For ease of notation, let us define $\gamma_s \triangleq -\frac{L_2^s - L_1^s}{c(P_2^s - P_1^s)} \geq 0$, $\gamma_r \triangleq -\frac{L_2^r - L_1^r}{c(P_2^r - P_1^r)} \geq 0$, $\gamma_s^0 \triangleq 1 + \gamma_s$, and $\gamma_r^0 \triangleq 1 + \gamma_r$. Then, according to (62) and (63), we have

$$\mu_2 - \mu_1 = a_2^s - a_1^s + c(b_1^s - b_2^s) + L_2^s - L_1^s \tag{68}$$

$$= (a_1^s - a_2^s) - c\gamma_s^0(b_1^s - b_2^s), \tag{69}$$

$$\nu_2 - \nu_1 = a_2^r - a_1^r + c(b_1^r - b_2^r) + L_2^r - L_1^r \tag{70}$$

$$= (a_1^r - a_2^r) - c\gamma_s^0(b_1^r - b_2^r). \tag{71}$$

Moreover, (67) can be equivalently expressed as

$$a_1^s b_2^s + a_1^r b_2^r \leq \frac{1}{4c} \left[(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2 \right]. \quad (72)$$

The concavity of R^{DF} in (P^s, P^r) suggests

$$a_1^s b_1^s + a_1^r b_1^r \leq 0, \quad a_2^s b_2^s + a_2^r b_2^r \leq 0. \quad (73)$$

If $a_i^s b_i^s \leq 0$ and $a_i^r b_i^r \leq 0$ ($i = 1, 2$), one can show Lemma 4 according to the arguments in [38]. However, rather than having $a_i^s b_i^s \leq 0$ or $a_i^r b_i^r \leq 0$, we only have (73). In order to handle the difficulty, we will discuss case by case to fully explore the structure of R^{DF} defined in (2). In particular, we proceed the remaining proof by breaking into three levels of cases:

- 1) Break into **Case 1-7** based on all combinations of the signs of $a_1^s b_1^s$, $a_1^r b_1^r$, $a_2^s b_2^s$, and $a_2^r b_2^r$.
- 2) In some cases, further break into subcases (I)-(IV) based on all combinations of the signs of $a_1^s b_2^s$ and $a_1^r b_2^r$.
- 3) In some subcases, further break into mini-cases (1)-(8) based on all combinations of \vec{P}_1 , \vec{P}_2 , and \vec{Q}^* lying in region ① or region ②, where region ① is defined as $R^{DF} = R_1$, i.e. $g^{s,d} P^s + g^{r,d} P^r \leq g^{s,r} P^s$, and region ② is defined as $R^{DF} = R_2$, i.e. $g^{s,d} P^s + g^{r,d} P^r \geq g^{s,r} P^s$.

We will explain in detail how to prove (72) for each subcase and mini-case in **Case 1**, and also in relative detail for **Case 2** to show that the proofs in **Case 1** and **Case 2** have similar logic. Since the techniques in **Case 3-7** are quite similar with that used in **Case 1**, we will omit most of the similar steps without repeating the same proof logic.

Case 1: When $a_1^s b_1^s \geq 0$, $a_1^r b_1^r \leq 0$, $a_2^s b_2^s \leq 0$, $a_2^r b_2^r \geq 0$.

(I) We first assume $a_1^s b_2^s \geq 0$, $a_1^r b_2^r \geq 0$, then we have $a_1^s a_2^s \leq 0$, $b_1^s b_2^s \geq 0$, $a_1^r a_2^r \geq 0$, $b_1^r b_2^r \leq 0$.

Now we further break into mini-cases:

(1) \vec{P}_1 is in region ①, \vec{P}_2 is in region ①, \vec{Q}^* is in region ①. Then,

$$\begin{aligned} a_1^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_1^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*}, \\ a_2^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_2^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)} - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}. \end{aligned}$$

If $b_1^s \geq 0$ and $b_1^r \leq 0$, then we must have $b_2^s \geq 0$ and $b_2^r \geq 0$ from $b_1^s b_2^s \geq 0$ and $b_1^r b_2^r \leq 0$, respectively. From the format of a_2^r , we further get $a_2^r \leq 0$, which contradicts $a_2^r b_2^r \geq 0$;

If $b_1^s \leq 0$ and $b_1^r \geq 0$, then $b_2^s \leq 0$ and $b_2^r \leq 0$. Further, from the format of a_2^r , we have $a_2^r \geq 0$, which contradicts $a_2^r b_2^r \geq 0$;

If $b_1^s \geq 0$ and $b_1^r \geq 0$, then from the format of a_1^s , we have $a_1^s \leq 0$, which contradicts $a_1^s b_1^s \geq 0$;

If $b_1^s \leq 0$ and $b_1^r \leq 0$, then $a_1^s \geq 0$, which contradicts $a_1^s b_1^s \geq 0$.

Thus, mini-case (1) is impossible under **Case 1**.

(2) \vec{P}_1 is in region ①, \vec{P}_2 is in region ②, \vec{Q}^* is in region ①. Then,

$$\begin{aligned} a_1^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_1^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*}, \\ a_2^s &= \frac{g^{s,r}}{1 + \frac{2}{\theta}g^{s,r}P_2^s} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_2^r &= - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}. \end{aligned}$$

Since $a_2^r \leq 0$, we get $b_2^r \leq 0$ from $a_2^r b_2^r \geq 0$ and $b_1^r \geq 0$ from $b_1^r b_2^r \leq 0$. Suppose $b_1^s \geq 0$, then from the format of a_1^s , we have $a_1^s \leq 0$ which contradicts $a_1^s b_1^s \geq 0$, so $b_1^s \leq 0$. By $b_1^s b_2^s \geq 0$, we

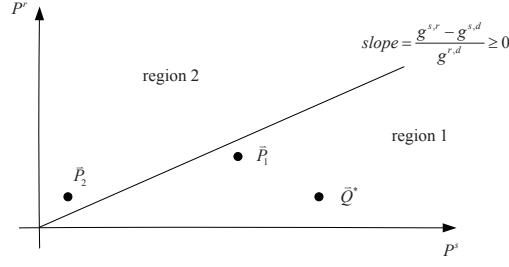


Fig. 7.

also have $b_2^s \leq 0$. With the above facts and Figure. 7, we have $P_2^s \leq P_1^s$, i.e., $b_2^s \leq b_1^s \leq 0$. It is apparent that $a_2^r \leq a_1^r \leq 0$, and $a_1^s \leq 0 \leq a_2^s$.

Suppose $P_1^s = 0$, then \vec{P}_1 is on the boundary of region ① and ②, and it belongs to mini-case (4) later. Without loss of generality, let $P_1^s \neq 0$, and then $L_1^s = 0$, $(a_2^s - a_1^s)(L_2^s - L_1^s) = (a_2^s - a_1^s)L_2^s \geq 0$. Similarly, $P_2^r \neq 0$, $L_2^r = 0$, and $(a_2^r - a_1^r)(L_2^r - L_1^r) = (a_1^r - a_2^r)L_1^r \geq 0$. Also, $a_1^s b_1^s + a_1^r b_1^r + a_2^s b_2^s + a_2^r b_2^r \leq 0$. Thus, we have

$$\begin{aligned}
& \frac{1}{4c} \left[(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2 \right] - a_1^s b_2^s - a_1^r b_2^r \\
&= \frac{1}{4c} (a_2^s - a_1^s)^2 + \frac{[c(b_1^s - b_2^s) + L_2^s - L_1^s]^2}{4c} + \\
& \quad \frac{1}{2c} (a_2^s - a_1^s)(L_2^s - L_1^s) - \frac{1}{2} (a_1^s b_2^s - a_2^s b_1^s) + \\
& \quad \frac{1}{4c} (a_2^r - a_1^r)^2 + \frac{[c(b_1^r - b_2^r) + L_2^r - L_1^r]^2}{4c} + \\
& \quad \frac{1}{2c} (a_2^r - a_1^r)(L_2^r - L_1^r) - \frac{1}{2} (a_1^r b_2^r - a_2^r b_1^r) - \\
& \quad \frac{1}{2} (a_1^s b_1^s + a_1^r b_1^r + a_2^s b_2^s + a_2^r b_2^r) \\
& \geq \frac{1}{4c} (a_2^r - a_1^r)^2 - \frac{1}{2} (a_1^s b_2^s - a_2^s b_1^s + a_1^r b_2^r - a_2^r b_1^r).
\end{aligned}$$

We want to choose c carefully such that the above term is nonnegative. Since \vec{Q}^* is feasible and Q^{s*} should be bounded by P_{\max}^s , we have $a_1^s b_2^s - a_2^s b_1^s \leq \frac{g^{s,d} Q^{s*}}{1 + \frac{2}{\theta}(g^{s,d} Q^{s*} + g^{r,d} Q^{r*})} + \frac{g^{s,r} Q^{s*}}{1 + \frac{2}{\theta} g^{s,r} P_{\max}^s} \leq \frac{1}{2} + g^{s,r} P_{\max}^s$. Similarly, $a_1^r b_2^r - a_2^r b_1^r \leq \frac{g^{s,r} - g^{r,d}}{g^{r,d}}$ and $(a_2^r - a_1^r)^2 = \left(\frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d} P_1^s + g^{r,d} P_1^r)} \right)^2 \geq$

$\left(\frac{g^{r,d}}{1+\frac{2}{\theta}g^{s,r}Q^{s*}}\right)^2$. Thus, if $\frac{1}{4c} \geq \frac{1}{2} \frac{\frac{1}{2}+g^{s,r}P_{\max}^s+\frac{g^{s,r}-g^{r,d}}{g^{r,d}}}{\left(\frac{g^{r,d}}{1+\frac{2}{\theta}g^{s,r}P_{\max}^s}\right)^2}$, i.e.,

$$c \leq \frac{\left(\frac{g^{r,d}}{1+\frac{2}{\theta}g^{s,r}P_{\max}^s}\right)^2}{1+2g^{s,r}P_{\max}^s+2\frac{g^{s,r}-g^{r,d}}{g^{r,d}}} \triangleq C_1,$$

we have $a_1^s b_2^s + a_1^r b_2^r \leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2]$.

Note that this direct argument is not general to other cases since $(a_2^r - a_1^r)^2$ may not have a positive lower bound if \vec{P}_1 and \vec{P}_2 are very close. So, breaking into cases is still necessary.

(3) \vec{P}_1 is in region ②, \vec{P}_2 is in region ①, \vec{Q}^* is in region ①.

$$\begin{aligned} a_1^s &= \frac{g^{s,r}}{1+\frac{2}{\theta}g^{s,r}P_1^s} - \frac{g^{s,d}}{1+\frac{2}{\theta}(g^{s,d}Q^{s*}+g^{r,d}Q^{r*})}, \\ a_1^r &= -\frac{g^{r,d}}{1+\frac{2}{\theta}(g^{s,d}Q^{s*}+g^{r,d}Q^{r*})}, \\ b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*}, \\ a_2^s &= \frac{g^{s,d}}{1+\frac{2}{\theta}(g^{s,d}P_2^s+g^{r,d}P_2^r)} - \frac{g^{s,d}}{1+\frac{2}{\theta}(g^{s,d}Q^{s*}+g^{r,d}Q^{r*})}, \\ a_2^r &= \frac{g^{r,d}}{1+\frac{2}{\theta}(g^{s,d}P_2^s+g^{r,d}P_2^r)} - \frac{g^{r,d}}{1+\frac{2}{\theta}(g^{s,d}Q^{s*}+g^{r,d}Q^{r*})}, \\ b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}. \end{aligned}$$

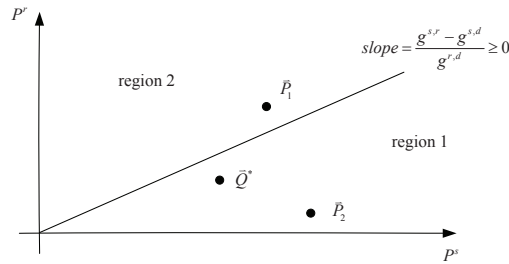


Fig. 8.

Since $a_1^r \leq 0$, we have $b_1^r \geq 0$ from $a_1^r b_1^r \leq 0$, and $a_2^r \geq 0$ from $a_1^r a_2^r \leq 0$. Suppose $b_1^s \leq 0$, from the format of a_1^s , we have $a_1^s \geq 0$, which contradicts $a_1^s b_1^s \geq 0$, so $b_1^s \geq 0$. Further, we have $a_1^s \geq 0$, $a_2^s \leq 0$, $b_2^s \geq 0$ by $a_1^s b_1^s \geq 0$, $a_1^s a_2^s \leq 0$ and $b_1^s b_2^s \geq 0$, respectively. Suppose $b_2^r \geq 0$, by

the format of a_2^r , $a_2^r \leq 0$, which contradicts $a_2^r b_2^r \geq 0$, so $b_2^r \leq 0$ and $a_2^r \leq 0$. If $P_1^s = 0$, we have $Q^{s*} = 0$ from b_1^s , which leads to triviality. Similar with mini-case (2), to avoid triviality, let $P_1^r \neq 0$, $P_2^s \neq 0$, then $L_1^s = L_1^r = L_2^s = 0$ and $(a_2^r - a_1^r)(L_2^r - L_1^r) = (a_2^r - a_1^r)L_2^r \geq 0$.

In this case, in order to apply the direct argument as in mini-case (2), we need a constant bound for P_1^s , P_1^r , P_2^s . Since $a_1^s = \frac{g^{s,r} - g^{s,d} + \frac{2}{\theta} g^{s,r} g^{s,d} (Q^{s*} - P_1^s) + \frac{2}{\theta} g^{s,r} g^{r,d} Q^{r*}}{(1 + \frac{2}{\theta} g^{s,r} P_1^s)(1 + \frac{2}{\theta} (g^{s,d} Q^{s*} + g^{r,d} Q^{r*}))} \geq 0$, we obtain

$$\begin{aligned} P_1^s &\leq \frac{\theta}{2g^{s,r}g^{s,d}}(g^{s,r} - g^{s,d} + \frac{2}{\theta}g^{s,r}g^{s,d}Q^{s*} + \frac{2}{\theta}g^{s,r}g^{r,d}Q^{r*}) \\ &\leq \frac{1}{2g^{s,r}g^{s,d}}(g^{s,r} - g^{s,d}) + P_{\max}^s + P_{\max}^r \triangleq X_1^s \end{aligned}$$

which is a constant bound for P_1^s . Note that

$$\begin{aligned} &a_1^s b_2^s + a_1^r b_2^r \\ &= \left[(a_1^s - a_2^s) - c\gamma_s^0 (b_1^s - b_2^s) \right] (b_2^s - b_1^s) + \\ &\quad a_2^s b_2^s + a_1^s b_1^s - a_2^s b_1^s - c\gamma_s^0 (b_2^s - b_1^s)^2 + \\ &\quad \left[(a_1^r - a_2^r) - c\gamma_r^0 (b_1^r - b_2^r) \right] (b_2^r - b_1^r) + \\ &\quad a_2^r b_2^r + a_1^r b_1^r - a_2^r b_1^r - c\gamma_r^0 (b_2^r - b_1^r)^2 \\ &= (\mu_2 - \mu_1)(b_2^s - b_1^s) - c\gamma_s^0 (b_2^s - b_1^s)^2 + \\ &\quad (\nu_2 - \nu_1)(b_2^r - b_1^r) - c\gamma_r^0 (b_2^r - b_1^r)^2 + \\ &\quad a_2^s b_2^s + a_1^s b_1^s - a_2^s b_1^s + a_2^r b_2^r + a_1^r b_1^r - a_2^r b_1^r \\ &\leq \frac{1}{4c\gamma_r^0}(\mu_2 - \mu_1)^2 + \frac{1}{4c\gamma_s^0}(\nu_2 - \nu_1)^2 + \\ &\quad a_2^s b_2^s + a_1^s b_1^s - a_2^s b_1^s + a_2^r b_2^r + a_1^r b_1^r - a_2^r b_1^r \end{aligned}$$

If $a_2^s b_2^s + a_1^s b_1^s - a_2^s b_1^s + a_2^r b_2^r + a_1^r b_1^r - a_2^r b_1^r \leq 0$, we are done, so we assume $a_2^s b_2^s + a_1^s b_1^s -$

$a_2^s b_1^s + a_2^r b_2^r + a_1^r b_1^r - a_2^r b_1^r \geq 0$. Recall that $a_2^s \leq a_1^s$, $a_1^r \leq a_2^r$ and $a_2^s b_2^s \leq 0$, we have

$$\begin{aligned}
& (a_1^s - a_2^s) b_1^s + a_2^r b_2^r \geq -a_2^s b_2^s + (a_2^r - a_1^r) b_1^r \geq -a_2^s b_2^s \\
& \geq \frac{(P_2^s - P_{\max}^s) g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d} P_{\max}^s + g^{r,d} P_{\max}^r)} - \frac{g^{s,d} P_2^s}{1 + \frac{2}{\theta}(g^{s,d} P_2^s + g^{r,d} P_2^r)} \\
& \geq \frac{g^{s,d} P_2^s}{1 + \frac{2}{\theta}(g^{s,d} P_{\max}^s + g^{r,d} P_{\max}^r)} - \\
& \quad \frac{g^{s,d} P_{\max}^s}{1 + \frac{2}{\theta}(g^{s,d} P_{\max}^s + g^{r,d} P_{\max}^r)} - \frac{\theta}{2}
\end{aligned}$$

Also, $(a_1^s - a_2^s) b_1^s + a_2^r b_2^r \leq g^{s,r} P_1^s + g^{r,d} P_{\max}^r = g^{s,r} X_1^s + g^{r,d} P_{\max}^r$. Combined with the above inequality, we have

$$\begin{aligned}
P_2^s & \leq \left[1 + \frac{2}{\theta}(g^{s,d} P_{\max}^s) \right] \times \left[g^{s,r} X_1^s + g^{r,d} P_{\max}^r \right. \\
& \quad \left. + \frac{1}{2} + \frac{g^{s,d} P_{\max}^s}{1 + \frac{2}{\theta}(g^{s,d} P_{\max}^s + g^{r,d} P_{\max}^r)} \right] \times \frac{1}{g^{s,d}} \triangleq X_2^s
\end{aligned}$$

which is a constant bound for P_2^s . Further,

$$\begin{aligned}
& (a_1^s - a_2^s) b_1^s + a_2^r b_2^r \geq (a_2^r - a_1^r) b_1^r \\
& \geq \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d} X_2^s + g^{r,d} P_{\max}^r)} P_1^r,
\end{aligned}$$

we then obtain

$$\begin{aligned}
P_1^r & \leq \left[1 + \frac{2}{\theta}(g^{s,d} X_2^s + g^{r,d} P_{\max}^r) \right] (g^{s,r} X_1^s + g^{r,d} P_{\max}^r) \frac{1}{g^{r,d}} \\
& \triangleq X_1^r
\end{aligned}$$

which is a constant bound for P_1^r . Now we can use the direct method as in mini-case (2), $a_1^s b_2^s - a_2^s b_1^s + a_1^r b_2^r - a_2^r b_1^r \leq g^{s,r} X_2^s + g^{s,d} X_1^s + g^{r,d} P_{\max}^r + g^{r,d} X_1^r$ and $(a_2^r - a_1^r)^2 \geq \left(\frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d} X_2^s + g^{r,d} X_1^r)} \right)^2$.

Thus, if $\frac{1}{4c} \geq \frac{g^{s,r} X_2^s + g^{s,d} X_1^s + g^{r,d} P_{\max}^r + g^{r,d} X_1^r}{2 \left(\frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d} X_2^s + g^{r,d} X_1^r)} \right)^2}$, i.e.,

$$c \leq \frac{\left(\frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d} X_2^s + g^{r,d} X_1^r)} \right)^2}{2(g^{s,r} X_2^s + g^{s,d} X_1^s + g^{r,d} P_{\max}^r + g^{r,d} X_1^r)} \triangleq C_2,$$

we have $a_1^s b_2^s + a_1^r b_2^r \leq \frac{1}{4c} \left[(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2 \right]$.

(4) \vec{P}_1 is in region ②, \vec{P}_2 is in region ②, \vec{Q}^* is in region ①.

$$\begin{aligned} a_1^s &= \frac{g^{s,r}}{1 + \frac{2}{\theta} g^{s,r} P_1^s} - \frac{g^{s,d}}{1 + \frac{2}{\theta} (g^{s,d} Q^{s*} + g^{r,d} Q^{r*})}, \\ a_1^r &= - \frac{g^{r,d}}{1 + \frac{2}{\theta} (g^{s,d} Q^{s*} + g^{r,d} Q^{r*})}, \\ b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*}, \\ a_2^s &= \frac{g^{s,r}}{1 + \frac{2}{\theta} g^{s,r} P_2^s} - \frac{g^{s,d}}{1 + \frac{2}{\theta} (g^{s,d} Q^{s*} + g^{r,d} Q^{r*})}, \\ a_2^r &= - \frac{g^{r,d}}{1 + \frac{2}{\theta} (g^{s,d} Q^{s*} + g^{r,d} Q^{r*})}, \\ b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}. \end{aligned}$$

Since $a_1^r \leq 0$, $a_2^r \leq 0$, then $b_1^r \geq 0$ and $b_2^r \leq 0$. Suppose $b_1^s \leq 0$, then $a_1^s \geq 0$ which contradicts $a_1^s b_1^s \geq 0$, so $b_1^s \geq 0$, $b_2^s \geq 0$ and $a_2^s \leq 0$. Now $b_2^s \geq 0$ and $b_2^r \leq 0$, then it is impossible to place \vec{P}_2 in region ② and \vec{Q}^* in region ① at the same time. So mini-case (4) is impossible under

Case 1.

(5) \vec{P}_1 is in region ①, \vec{P}_2 is in region ①, \vec{Q}^* is in region ②.

$$\begin{aligned} a_1^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta} (g^{s,d} P_1^s + g^{r,d} P_1^r)} - \frac{g^{s,r}}{1 + \frac{2}{\theta} g^{s,r} Q^{s*}}, \\ a_1^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta} (g^{s,d} P_1^s + g^{r,d} P_1^r)}, \\ b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*}, \\ a_2^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta} (g^{s,d} P_2^s + g^{r,d} P_2^r)} - \frac{g^{s,r}}{1 + \frac{2}{\theta} g^{s,r} Q^{s*}}, \\ a_2^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta} (g^{s,d} P_2^s + g^{r,d} P_2^r)}, \\ b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}. \end{aligned}$$

Since $a_1^r \geq 0$ and $a_2^r \geq 0$, then $b_1^r \leq 0$ and $b_2^r \geq 0$. Suppose $b_1^s \geq 0$, then $a_1^s \leq 0$ which contradicts $a_1^s b_1^s \geq 0$, so $b_1^s \leq 0$, $b_2^s \leq 0$ and $a_2^s \geq 0$. Now $b_2^s \leq 0$ and $b_2^r \geq 0$, it is impossible to place \vec{P}_2 in region ① and \vec{Q}^* in region ②. So, mini-case (5) is impossible under **Case 1.**

(6) \vec{P}_1 is in region ①, \vec{P}_2 is in region ②, \vec{Q}^* is in region ②.

In this case $a_2^r = 0$. By using the result $a_1^s a_2^s \leq 0$, we have

$$\begin{aligned}
& a_1^s b_2^s + a_1^r b_2^r \\
& \leq a_1^s b_2^s - \frac{a_1^s a_2^s}{c\gamma_s^0} + a_1^r b_2^r - \frac{a_1^r a_2^r}{c\gamma_r^0} \\
& = \frac{1}{c\gamma_s^0} \left\{ [(a_1^s - a_2^s) - c\gamma_s^0(b_1^s - b_2^s)] a_1^s + (c\gamma_s^0 b_1^s - a_1^s) a_1^s \right\} + \\
& \quad \frac{1}{c\gamma_r^0} \left\{ [(a_1^r - a_2^r) - c\gamma_r^0(b_1^r - b_2^r)] a_1^r + (c\gamma_r^0 b_1^r - a_1^r) a_1^r \right\} \\
& \leq \frac{1}{c\gamma_s^0} \left\{ (\mu_2 - \mu_1) a_1^s - (a_1^s)^2 \right\} \\
& \quad + \frac{1}{c\gamma_r^0} \left\{ (\nu_2 - \nu_1) a_1^r - (a_1^r)^2 \right\} + (a_1^s b_1^s + a_1^r b_1^r) \\
& \leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2],
\end{aligned}$$

where in the last step, we have used $\gamma_s^0 = 1 + \gamma_s \geq 1$, $\gamma_r^0 = 1 + \gamma_r \geq 1$, and $a_1^s b_1^s + a_1^r b_1^r \leq 0$.

(7) \vec{P}_1 is in region ②, \vec{P}_2 is in region ①, \vec{Q}^* is in region ②.

In this case $a_1^r = 0$, then from $a_1^s b_1^s + a_1^r b_1^r \leq 0$, we have $a_1^s b_1^s \leq 0$. Further, since $a_1^s b_2^s \geq 0$, we have $b_1^s b_2^s \leq 0$. In view of the result $b_1^r b_2^r \leq 0$, we then have

$$\begin{aligned}
& a_1^s b_2^s + a_1^r b_2^r \\
& \leq a_1^s b_2^s - c\gamma_s^0 b_1^s b_2^s + a_1^r b_2^r - c\gamma_r^0 b_1^r b_2^r \\
& = [(a_1^s - a_2^s) - c\gamma_s^0(b_1^s - b_2^s)] b_2^s \\
& \quad + [(a_1^r - a_2^r) - c\gamma_r^0(b_1^r - b_2^r)] b_2^r \\
& \quad + (a_2^s - c\gamma_s^0 b_2^s) b_2^s + (a_2^r - c\gamma_r^0 b_2^r) b_2^r \\
& \leq (\mu_2 - \mu_1) b_2^s - c\gamma_s^0 (b_2^s)^2 + \\
& \quad (\nu_2 - \nu_1) b_2^r - c\gamma_r^0 (b_2^r)^2 + (a_2^s b_2^s + a_2^r b_2^r) \\
& \leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2].
\end{aligned}$$

(8) \vec{P}_1 is in region ②, \vec{P}_2 is in region ②, \vec{Q}^* is in region ②.

$a_1^r b_1^r = 0$, $a_2^r b_2^r = 0$, $a_1^s b_1^s \leq 0$, $a_2^s b_2^s \leq 0$, then $a_1^s b_2^s + a_1^r b_2^r = a_1^s b_2^s$, and the techniques in [38] applies.

(II) If $a_1^s b_2^s \leq 0$ and $a_1^r b_2^r \leq 0$, then is is trivial.

(III) If $a_1^s b_2^s > 0$ and $a_1^r b_2^r < 0$, then let $\gamma^s \triangleq -\frac{a_2^s}{c\gamma_s^0 b_2^s} \geq 0$ and $\gamma^r \triangleq -\frac{a_2^r}{c\gamma_r^0 b_2^r} \leq 0$.

$$\begin{aligned} a_1^s b_2^s + a_1^r b_2^r &\leq (1 + \gamma^s) a_1^s b_2^s + (1 + \gamma^r) a_1^r b_2^r \\ &= \frac{1}{c\gamma_s^0} \left\{ [(a_1^s - a_2^s) - c\gamma_s^0 (b_1^s - b_2^s)] a_1^s + (c\gamma_s^0 b_1^s - a_1^s) a_1^s \right\} + \\ &\quad \frac{1}{c\gamma_r^0} \left\{ [(a_1^r - a_2^r) - c\gamma_r^0 (b_1^r - b_2^r)] a_1^r + (c\gamma_r^0 b_1^r - a_1^r) a_1^r \right\} \\ &\leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2]. \end{aligned}$$

(IV) If $a_1^s b_2^s < 0$ and $a_1^r b_2^r > 0$, it can be dealt with similarly as above.

Case 2: When $a_1^s b_1^s \leq 0$, $a_1^r b_1^r \geq 0$, $a_2^s b_2^s \geq 0$, $a_2^r b_2^r \leq 0$.

(I) If $a_1^s b_2^s \geq 0$ and $a_1^r b_2^r \geq 0$, then we have $a_1^s a_2^s \geq 0$, $b_1^s b_2^s \leq 0$, $a_1^r a_2^r \leq 0$, $b_1^r b_2^r \geq 0$. Now we further break into mini-cases:

(1) \vec{P}_1 is in region ①, \vec{P}_2 is in region ①, \vec{Q}^* is in region ①.

$$\begin{aligned} a_1^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_1^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*} \\ a_2^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_2^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)} - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}. \end{aligned}$$

If $b_1^s \geq 0$ and $b_1^r \leq 0$, then $b_2^s \leq 0$ and $b_2^r \leq 0$, and we further have $a_2^s \geq 0$ and $a_2^r \geq 0$, which contradicts $a_2^s b_2^s \geq 0$;

If $b_1^s \leq 0$ and $b_1^r \geq 0$, then $b_2^s \geq 0$ and $b_2^r \geq 0$, and we further have $a_2^s \leq 0$ and $a_2^r \leq 0$, which contradicts $a_2^s b_2^s \geq 0$;

If $b_1^s \geq 0$ and $b_1^r \geq 0$, then $a_1^s \leq 0$ and $a_1^r \leq 0$, which contradicts $a_1^r b_1^r \geq 0$;

If $b_1^s \leq 0$ and $b_1^r \leq 0$, then $a_1^s \geq 0$ and $a_1^r \geq 0$, which contradicts $a_1^r b_1^r \geq 0$.

So mini-case (1) is impossible under **Case 2**.

(2) \vec{P}_1 is in region ①, \vec{P}_2 is in region ②, \vec{Q}^* is in region ①.

$$\begin{aligned}
a_1^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\
a_1^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\
b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*} \\
a_2^s &= \frac{g^{s,r}}{1 + \frac{2}{\theta}g^{s,r}P_2^s} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\
a_2^r &= -\frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\
b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}.
\end{aligned}$$

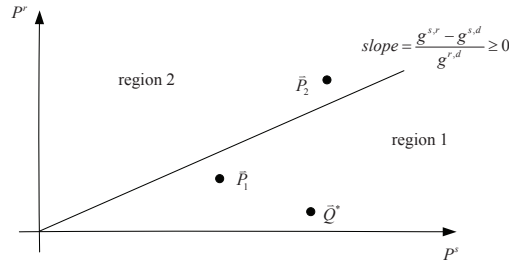


Fig. 9.

Since $a_2^r \leq 0$, we get $b_2^r \geq 0$ by $a_2^r b_2^r \leq 0$ and $b_1^r \geq 0$ by $b_1^r b_2^r \geq 0$. Suppose $b_1^s \geq 0$, then $a_1^s \leq 0$ and $a_1^r \leq 0$, which contradicts $a_1^r b_1^r \geq 0$, so $b_1^s \leq 0$. Then, $b_2^s \geq 0$, $a_1^s \geq 0$, $a_1^r \geq 0$, $a_2^s \geq 0$. Without loss of generality, let $P_1^s > 0$, $P_1^r > 0$, $P_2^s > 0$, $P_2^r > 0$, so $L_1^s = L_2^s = L_1^r = L_2^r = 0$. Note that $P_1^s \leq Q^{s*} \leq P_{\max}^s$ and $P_1^r \leq \frac{g^{s,r} - g^{s,d}}{g^{r,d}} P_1^s = \frac{g^{s,r} - g^{s,d}}{g^{r,d}} P_{\max}^s$. Since $a_2^s = \frac{g^{s,r} - g^{s,d} + \frac{2}{\theta} g^{s,r} g^{s,d} (Q^{s*} - P_2^s) + \frac{2}{\theta} g^{s,r} g^{r,d} Q^{r*}}{(1 + \frac{2}{\theta} g^{s,r} P_2^s)(1 + \frac{2}{\theta} (g^{s,d} Q^{s*} + g^{r,d} Q^{r*}))} \geq 0$, we obtain

$$\begin{aligned}
P_2^s &\leq \frac{\theta}{2g^{s,r}g^{s,d}} \left(g^{s,r} - g^{s,d} + \frac{2}{\theta} g^{s,r} g^{s,d} Q^{s*} + \frac{2}{\theta} g^{s,r} g^{r,d} Q^{r*} \right) \\
&\leq \frac{1}{2g^{s,r}g^{s,d}} (g^{s,r} - g^{s,d}) + P_{\max}^s + P_{\max}^r \triangleq X_1^s
\end{aligned}$$

which is a constant bound for P_2^s . Use the similar idea as in **Case 1**. Without loss of generality, assume $a_2^s b_2^s + a_1^s b_1^s - a_2^s b_1^s + a_1^r b_2^r + a_1^r b_1^r - a_2^r b_1^r \geq 0$, then $a_2^s b_2^s - a_2^s b_1^s + a_1^r b_1^r - a_2^r b_1^r \geq -a_1^s b_1^s - a_2^r b_2^r \geq -a_2^r b_2^r \geq \frac{g^{r,d}(P_2^r - P_{\max}^r)}{1 + \frac{2}{\theta}(g^{s,d}P_{\max}^s + g^{r,d}P_{\max}^r)}$. Also, $a_2^s(b_2^s - b_1^s) + (a_1^r - a_2^r)b_1^r \leq \frac{\theta}{2} + g^{r,d} \frac{g^{s,r} - g^{s,d}}{g^{r,d}} P_{\max}^s =$

$\frac{\theta}{2} + (g^{s,r} - g^{s,d})P_{\max}^s$, then

$$\begin{aligned} P_2^r &\leq \frac{1}{g^{r,d}} \left[\frac{1}{2} + (g^{s,r} - g^{s,d})P_{\max}^s \right] \times \\ &\quad \left[1 + \frac{2}{\theta}(g^{s,d}P_{\max}^s + g^{r,d}P_{\max}^r) \right] + P_{\max}^r \\ &\triangleq X_2^r \end{aligned}$$

which is a constant bound for P_2^r . Thus, if $\frac{1}{4c} \geq \frac{1}{2} \frac{g^{s,d}X_2^s + g^{s,r}P_{\max}^s + g^{r,d}X_2^r + (g^{s,r} - g^{s,d})P_{\max}^s}{(\frac{g^{r,d}}{1 + \frac{2}{\theta}g^{s,r}P_{\max}^s})^2} \triangleq C_3$.

(3) \vec{P}_1 is in region ②, \vec{P}_2 is in region ①, \vec{Q}^* is in region ①.

$$\begin{aligned} a_1^s &= \frac{g^{s,r}}{1 + \frac{2}{\theta}g^{s,r}P_1^s} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_1^r &= - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_1^s &= P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*} \\ a_2^s &= \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_2^r &= \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)} - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_2^s &= P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}. \end{aligned}$$

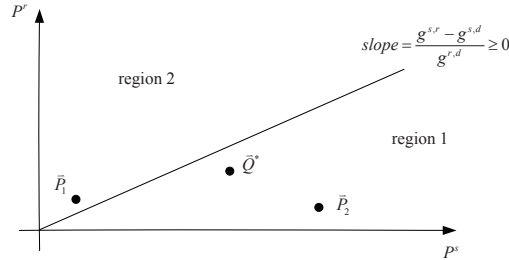


Fig. 10.

Since $a_1^r \leq 0$, we get $b_1^r \leq 0$ and $b_2^r \leq 0$. Suppose $b_2^s \leq 0$, then $a_2^s \geq 0$ and $a_2^r \geq 0$, which contradicts $a_2^s b_2^s \geq 0$, so $b_2^s \geq 0$, $b_1^s \leq 0$, $a_1^s \geq 0$, $a_2^s \geq 0$, $a_2^r \geq 0$. Without loss of generality, let $P_1^r > 0$ and $P_2^s > 0$, so $L_1^r = 0$ and $L_2^s = 0$. Further, $P_1^s \leq P_2^s$, then $a_2^s \leq a_1^s$, so $(a_2^s - a_1^s)(-L_1^s) \geq 0$ and $(a_2^r - a_1^r)L_2^r \geq 0$. Note that $P_1^s \leq Q^{s*} \leq P_{\max}^s$, $P_1^r \leq Q^{r*} \leq P_{\max}^r$, $P_2^s \leq Q^{r*} \leq P_{\max}^r$, and

$$a_2^s = g^{s,d} \frac{\frac{2}{\theta}[g^{s,d}(Q^{s*}-P_2^s)+g^{r,d}(Q^{r*}-P_2^r)]}{[1+\frac{2}{\theta}(g^{s,d}P_2^s+g^{r,d}P_2^r)][1+\frac{2}{\theta}(g^{s,d}Q^{s*}+g^{r,d}Q^{r*})]} \geq 0, \text{ then } P_2^s \leq Q^{s*} + \frac{g^{r,d}}{g^{s,d}}Q^{r*} = P_{\max}^s + \frac{g^{r,d}}{g^{s,d}}P_{\max}^r.$$

$$\text{Thus, if } \frac{1}{4c} \geq \frac{1}{2} \frac{g^{s,r}(P_{\max}^s + \frac{g^{r,d}}{g^{s,d}}P_{\max}^r) + g^{s,d}P_{\max}^s + g^{r,d}P_{\max}^r + g^{r,d}P_{\max}^r}{\left(\frac{g^{r,d}}{1+\frac{2}{\theta}[g^{s,d}(P_{\max}^s + \frac{g^{r,d}}{g^{s,d}}P_{\max}^r) + g^{r,d}P_{\max}^r]} \right)^2}, \text{ i.e.,}$$

$$c \leq \frac{1}{2} \frac{\left(\frac{g^{r,d}}{1+\frac{2}{\theta}(g^{s,d}P_{\max}^s + 2g^{r,d}P_{\max}^r)} \right)^2}{(g^{s,r} + g^{s,d})P_{\max}^s + (\frac{g^{s,r}g^{r,d}}{g^{s,d}} + 2g^{r,d})P_{\max}^r} \\ \triangleq C_4$$

(4) \vec{P}_1 is in region ②, \vec{P}_2 is in region ②, \vec{Q}^* is in region ①.

$$a_1^s = \frac{g^{s,r}}{1 + \frac{2}{\theta}g^{s,r}P_1^s} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_1^r = - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_1^s = P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*} \\ a_2^s = \frac{g^{s,r}}{1 + \frac{2}{\theta}g^{s,r}P_2^s} - \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ a_2^r = - \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}Q^{s*} + g^{r,d}Q^{r*})}, \\ b_2^s = P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}.$$

$a_1^r = a_2^r < 0$, this contradicts $a_1^r a_2^r \leq 0$. So mini-case (4) is impossible under **Case 2**.

(5) \vec{P}_1 is in region ①, \vec{P}_2 is in region ①, \vec{Q}^* is in region ②.

$$a_1^s = \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)} - \frac{g^{s,r}}{1 + \frac{2}{\theta}g^{s,r}Q^{s*}}, \\ a_1^r = \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_1^s + g^{r,d}P_1^r)}, \\ b_1^s = P_1^s - Q^{s*}, \quad b_1^r = P_1^r - Q^{r*} \\ a_2^s = \frac{g^{s,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)} - \frac{g^{s,r}}{1 + \frac{2}{\theta}g^{s,r}Q^{s*}}, \\ a_2^r = \frac{g^{r,d}}{1 + \frac{2}{\theta}(g^{s,d}P_2^s + g^{r,d}P_2^r)}, \\ b_2^s = P_2^s - Q^{s*}, \quad b_2^r = P_2^r - Q^{r*}.$$

$a_1^r \geq 0, a_2^r \geq 0$, this contradicts $a_1^r a_2^r \leq 0$. So mini-case (5) is impossible under **Case 2**.

(6),(7),(8) are all similar as in **Case 1**.

(II) If $a_1^s b_2^s \leq 0$ and $a_1^r b_2^r \leq 0$, it is trivial;

(III) If $a_1^s b_2^s \geq 0$ and $a_1^r b_2^r \leq 0$, then let $\gamma^s \triangleq -\frac{c\gamma_s^0 b_1^s}{a_1^s} \geq 0$ and $\gamma^r \triangleq -\frac{c\gamma_r^0 b_1^r}{a_1^r} \leq 0$ and similar argument as in **Case 1** follows;

(IV) If $a_1^s b_2^s \leq 0$ and $a_1^r b_2^r \geq 0$, it is similar as above.

Case 3: When $a_1^s b_1^s \leq 0, a_1^r b_1^r \geq 0, a_2^s b_2^s \leq 0, a_2^r b_2^r \geq 0$.

We only prove for the subcase when $a_1^s b_2^s > 0$ and $a_1^r b_2^r > 0$. The same proof ideas as in **Case 1** can be applied in all other subcases.

Let $\gamma^s \triangleq \frac{a_2^s b_1^s}{a_1^s b_2^s} \geq 0$ and $\gamma^r \triangleq \frac{a_2^r b_1^r}{a_1^r b_2^r} \geq 0$, then

$$\begin{aligned}
& a_1^s b_2^s + a_1^r b_2^r \\
& \leq (1 + \gamma^s) a_1^s b_2^s + (1 + \gamma^r) a_1^r b_2^r \\
& = [(a_1^s - a_2^s) - c\gamma_s^0(b_1^s - b_2^s)](b_2^s - b_1^s) \\
& \quad + [(a_1^r - a_2^r) - c\gamma_r^0(b_1^r - b_2^r)](b_2^r - b_1^r) \\
& \quad + (a_1^s b_1^s + a_2^s b_2^s) + (a_1^r b_1^r + a_2^r b_2^r) \\
& \quad - c\gamma_s^0(b_2^s - b_1^s)^2 - c\gamma_r^0(b_2^r - b_1^r)^2 \\
& \leq (\mu_2 - \mu_1)(b_2^s - b_1^s) - c\gamma_s^0(b_2^s - b_1^s)^2 \\
& \quad + (\nu_2 - \nu_1)(b_2^r - b_1^r) - c\gamma_r^0(b_2^r - b_1^r)^2 \\
& \quad + (a_1^s b_1^s + a_1^r b_1^r) + (a_2^s b_2^s + a_2^r b_2^r) \\
& \leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2].
\end{aligned}$$

Case 4: When $a_1^s b_1^s \geq 0, a_1^r b_1^r \leq 0, a_2^s b_2^s \geq 0, a_2^r b_2^r \leq 0$.

The similar argument as in **Case 3** can be applied here.

Case 5: When $a_1^s b_1^s \leq 0, a_1^r b_1^r \leq 0$.

We only prove for the subcase when $a_1^s b_2^s > 0$ and $a_1^r b_2^r > 0$. The same proof ideas as in **Case 1**

can be applied in all other subcases. Let $\gamma^s \triangleq -\frac{c\gamma_s^0 b_1^s}{a_1^s} \geq 0$, $\gamma^r \triangleq -\frac{c\gamma_r^0 b_1^r}{a_1^r} \geq 0$, then

$$\begin{aligned}
& a_1^s b_2^s + a_1^r b_2^r \\
& \leq (1 + \gamma^s) a_1^s b_2^s + (1 + \gamma^r) a_1^r b_2^r \\
& = [(a_1^s - a_2^s) - c\gamma_s^0 (b_1^s - b_2^s)] b_2^s \\
& \quad + [(a_1^r - a_2^r) - c\gamma_r^0 (b_1^r - b_2^r)] b_2^r \\
& \quad + (a_2^s - c\gamma_s^0 b_2^s) b_2^s + (a_2^r - c\gamma_r^0 b_2^r) b_2^r \\
& \leq (\mu_2 - \mu_1) b_2^s - c\gamma_s^0 (b_2^s)^2 + (\nu_2 - \nu_1) b_2^r - c\gamma_r^0 (b_2^r)^2 + \\
& \quad (a_2^s b_2^s + a_2^r b_2^r) \\
& \leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2].
\end{aligned}$$

Case 6: $a_2^s b_2^s \leq 0$, $a_2^r b_2^r \leq 0$.

We only prove for the subcase when $a_1^s b_2^s > 0$ and $a_1^r b_2^r > 0$. The same proof ideas as in **Case 1** can be applied in all other subcases. Let $\gamma^s \triangleq -\frac{a_2^s}{c\gamma_s^0 b_2^s} \geq 0$, $\gamma^r \triangleq -\frac{a_2^r}{c\gamma_r^0 b_2^r} \geq 0$, then

$$\begin{aligned}
& a_1^s b_2^s + a_1^r b_2^r \\
& \leq (1 + \gamma^s) a_1^s b_2^s + (1 + \gamma^r) a_1^r b_2^r \\
& = \frac{1}{c\gamma_s^0} \left\{ [(a_1^s - a_2^s) - c\gamma_s^0 (b_1^s - b_2^s)] a_1^s + (c\gamma_s^0 b_1^s - a_1^s) a_1^s \right\} + \\
& \quad \frac{1}{c\gamma_r^0} \left\{ [(a_1^r - a_2^r) - c\gamma_r^0 (b_1^r - b_2^r)] a_1^r + (c\gamma_r^0 b_1^r - a_1^r) a_1^r \right\} \\
& \leq \frac{1}{c\gamma_s^0} \left\{ (\mu_2 - \mu_1) a_1^s - (a_1^s)^2 \right\} + \\
& \quad \frac{1}{c\gamma_r^0} \left\{ (\nu_2 - \nu_1) a_1^r - (a_1^r)^2 \right\} + (a_1^s b_1^s + a_1^r b_1^r) \\
& \leq \frac{1}{4c} [(\mu_2 - \mu_1)^2 + (\nu_2 - \nu_1)^2].
\end{aligned}$$

Case 7: When $a_1^s b_1^s > 0$, $a_1^r b_1^r \geq 0$; or $a_2^s b_2^s > 0$, $a_2^r b_2^r \geq 0$. This case can not happen, otherwise, it contradicts (73). ■